1. Design of rectangular beam with tension reinforcement (singly reinforcement) with pre-specified dimensions (b,h):

- In design problem, usually the beam **span**, **beam dimensions (b, h), dead** and **live loads** are given.
- Material strength (*fc`* and *fy*) are given.
- Then the main unknown in the design process is the reinforcement detail that can be summarized as follows:
  - **i. Area of reinforcement**
  - **ii. Number and diameters of rebars**
  - **iii. Number of layers that required for these rebars**

Design Procedure

1. Compute required factored applied moment **Mu** based on given load (dead and live loads).
2. Compute the effective depth (d)

   $$\mathbf{d_{\text{for one layer}} = h\text{-}cover\text{-}stirrups\text{-}\frac{\textit{bar diameter}}{2}}$$

   $$\mathbf{d_{\text{for two layer}} = h\text{-}cover\text{-}stirrups\text{-}bar\ diameter\text{-}\frac{\textit{spacing between layers}}{2}}$$

   - Based on above two relations, the designer must assume values for following items to be able to compute the effective depth "d".
   - The ACI requirements for beams cover are **<u>40 mm.</u>**
   - **<u>Diameter 16mm to 25mm</u>** is usually used in for beam reinforcement.
   - **<u>Diameter 10mm and 12mm</u>** is usually used in stirrups.
3. Compute ρ_{max}, ρ_{min}

   $$\rho_{max} = 0.85\beta_1 \frac{fc`}{fy} \frac{\in u}{\in u + 0.004} \quad \text{and } \in u = 0.003$$

   $$\rho_{min} \frac{1.4}{fy} \text{ or } \frac{\sqrt{fc`}}{4fy}$$
4. Compute ρ

   $$\rho = \frac{1}{m}\left(1 - \sqrt{1 - \frac{2Rm}{fy}}\right)$$

   Where:

   $$m = \frac{fy}{0.85fc`} \quad , \quad R = \frac{M_u * 10^6}{\varphi b d^2}$$

   kN.m

   Both of **b** and **d** are in mm

   Assume φ=0.9

- If $\rho > \rho_{max}$

  Then the designer must increase one or both beam dimensions (b and d) and resolve the problem from step 2, or design the beam as double reinforcement as will be discussing in next topics.

  If $\rho < \rho_{min}$ use $\rho = \rho_{min}$

- If $\rho_{min} \leq \rho \leq \rho_{max}$ O.K

5. Compute the required steel area (As):

   As= $\rho * b * d$

6. Compute required Number of rebars:

   No. of rebars= $\frac{As}{Abar}$

   - Round up the required rebars number to nearest larger integer number then find the new $As_{provided}$.

7. Check if the available width "b" is adequate to put the rebars in a single layer:

   $b_{required}$ = 2 x cover+2 x stirrups diameter + No. of rebars x bar diameter + (No. of rebars-1) x spacing between rebars

If

$b_{required} > b_{aviable}$

Then reinforcement cannot be put in a single layer. If your calculations have be based on assumption of single layer, then you must retain to **setup 2** and recalculate "d" based on two reinforcement layers.

S_{clear} = larger [25 mm, d_b]

8. Check the assumption of φ=0.9

   $a = \frac{As * fy}{0.85fc` * b}$

   $c = \frac{a}{\beta 1}$

   $\in t = \frac{dt - c}{c} \in u$ where: $\in u = 0.003$

- If $\in t \geq 0.005$, then φ=0.9
- If $\in t < 0.005$ then

  $\varphi = 0.483 + 83.3 * \in t$

  And retain to step 4

9. Draw final detailed section

Minimum depth for beams

ACI code provides minimum thickness of beams, the purpose of such limitations is to prevent deflections of such magnitudes as would interfere with the use of or cause the injury to the structures.

Table 9.3.1.1—Minimum depth of nonprestressed beams

Support condition	Minimum h[1]
Simply supported	$\ell/16$
One end continuous	$\ell/18.5$
Both ends continuous	$\ell/21$
Cantilever	$\ell/8$

[1]Expressions applicable for normalweight concrete and Grade 420 reinforcement. For other cases, minimum h shall be modified in accordance with 9.3.1.1.1 through 9.3.1.1.3, as appropriate.

- For fy other than 420 Mpa, the expression in Table 9.3.1.1 must multiplied by (0.4+fy/700)
- The most economical beam sections are usually obtained for shorter beams (up to 6m or 8m), when the ratio of **d** to **b** is in the range of 1.5 to 2.

Example: Find the dimensions of simply supported beam with a length equal to 6 m, use fy=420 Mpa , use h/b=1.5

Solution:

According to ACI Table 9.3.1.1

$$h=\frac{\ell}{16}=\frac{6000}{16}=375 \text{ mm} \approx 400 \text{ mm}$$

$h/b=1.5$

$400/b=1.5$

$b=266.67 \text{ mm} \approx 280 \text{ mm}$

Table 9.3.1.1—Minimum depth of nonprestressed beams

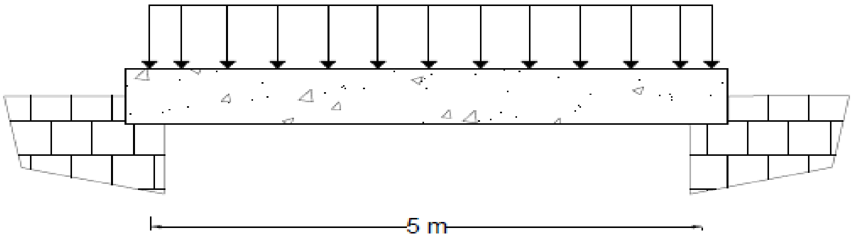
Support condition	Minimum h[1]
Simply supported	$\ell/16$
One end continuous	$\ell/18.5$
Both ends continuous	$\ell/21$
Cantilever	$\ell/8$

[1]Expressions applicable for normalweight concrete and Grade 420 reinforcement. For other cases, minimum h shall be modified in accordance with 9.3.1.1.1 through 9.3.1.1.3, as appropriate.

Example1: Design a simply supported rectangular reinforced concrete beam shown in Figure below.

Assume that the designer intends to use:

- Concrete of $fc`$ =30 Mpa.
- Steel fy = 400 Mpa.
- A width of 300 mm and a height of 430 mm.
- Rebar diameter 25mm for longitudinal reinforcement.
- Rebar diameter 10mm for stirrups.
- Single layer of reinforcement.
- W_D**(without self-weight)**=10 kN/m and W_L=25.77 kN/m



Solution:

1. Compute required factored applied moment **Mu**

$W_{selfweight}$=0.43*0.3*24=3.1 kN/m

W_D=3.1+10=13.1 kN/m

WL=25.77 kN/m

$W_U=1.2W_D+1.6W_L$

W_U=1.2*13.1+1.6*25.77

W_U=56.95 kN/m

$M_u=\frac{Wu\ell^2}{8}=\frac{56.95*5^2}{8}=177.96$ kN.m

2. Compute the effective depth (d)

$\mathbf{d}_{\textbf{for one layer}}$**=h-cover-stirrups-**$\frac{\boldsymbol{bar\ diameter}}{\mathbf{2}}$

$d=430-40-10-\frac{25}{2}=368$ mm

3. Compute ρ_{max}, ρ_{min}

$\rho_{max}=0.85\beta_1\frac{fc`}{fy}\frac{\epsilon u}{\epsilon u+0.004}$ and $\epsilon\ u=0.003$

$\beta_1=0.85-\frac{0.05(30-28)}{7}=0.836>0.65$ O.k

Table 22.2.2.4.3—Values of β_1 for equivalent rectangular concrete stress distribution

f_c', MPa	β_1	
$17 \le f_c' \le 28$	0.85	(a)
$28 < f_c' < 55$	$0.85-\frac{0.05(f_c'-28)}{7}$	(b)
$f_c' \ge 55$	0.65	(c)

$\rho_{max}=0.85*0.836*\frac{30}{400}\frac{0.003}{0.003+0.004}=22.8*10^{-3}$

$\rho_{min}\frac{1.4}{fy}=\frac{1.4}{400}=3.5*10^{-3}$

4. Compute ρ

$$\rho=\frac{1}{m}\left(1-\sqrt{1-\frac{2Rm}{fy}}\right)$$

$$m=\frac{fy}{0.85fc`}=\frac{400}{0.85*30}=15.6$$

$$R=\frac{M_u*10^6}{\varphi bd^2}=\frac{177.96*10^6}{0.9*300*368^2}=4.86$$

$$\rho=\frac{1}{15.6}\left(1-\sqrt{1-\frac{2*4.86*15.6}{400}}\right)=13.6*10^{-3}$$

$\rho_{min}=3.5*10^{-3}<\rho=13.6*10^{-3}<\rho_{max}=22.8*10^{-3}$ O.K

5. Compute the required steel area (As):

$As=\rho*b*d=13.6*10^{-3}*300*368=1501.44\ mm^2$

6. Compute required Number of rebars:

$$\text{No. of rebars}=\frac{As}{Abar}=\frac{1501.44}{\frac{\pi}{4}*25^2}=\frac{1501.44}{491}=3.1\approx 4$$

Try $4\varphi25$mm.

$As_{provided}=4*491=1964\ mm^2$

7. Check if the available width “b” is adequate to put the rebars in a single layer:

$b_{required}$=2 x cover+2 x stirrups diameter + No. of rebars x bar diameter + (No. of rebars-1) x spacing between rebars

$b_{required}$=2*40+2*10+4*25+3*25=275 mm < 300 mm O.K

8. Check the assumption of $\varphi=0.9$

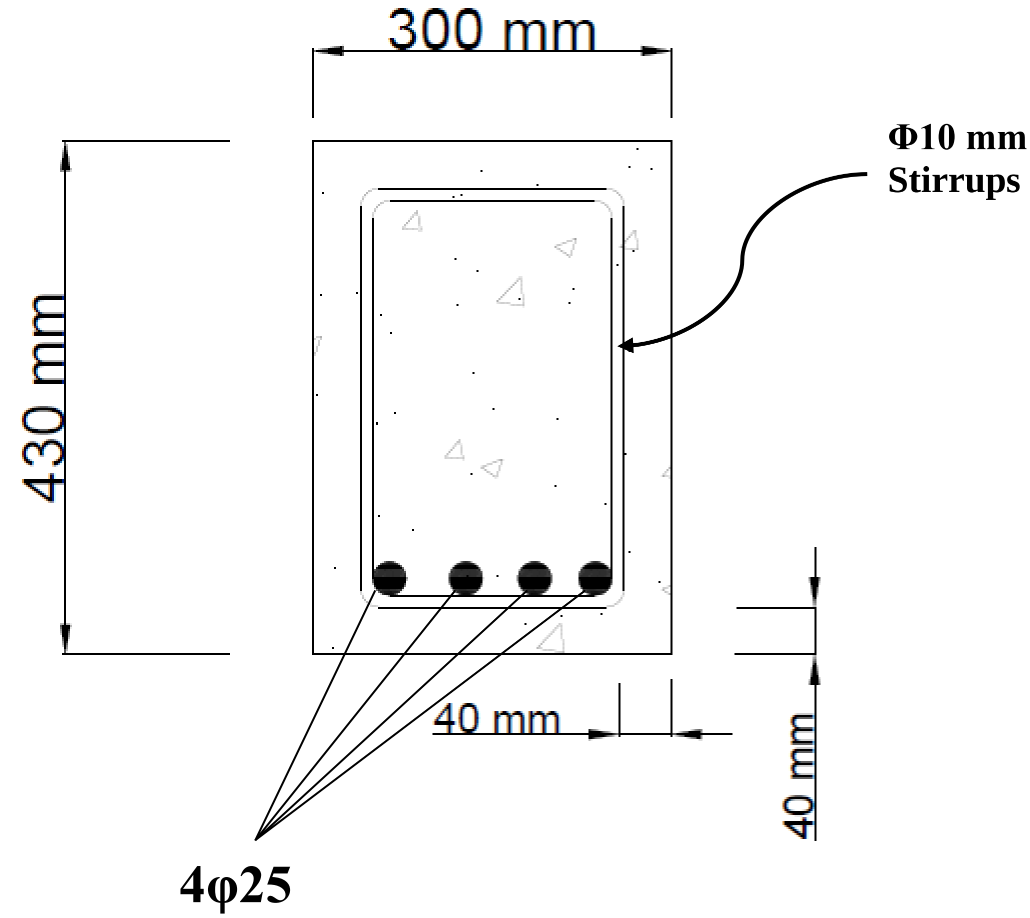
$$a=\frac{As*fy}{0.85fc`*b}=\frac{1964*400}{0.85*30*300}=102\ mm$$

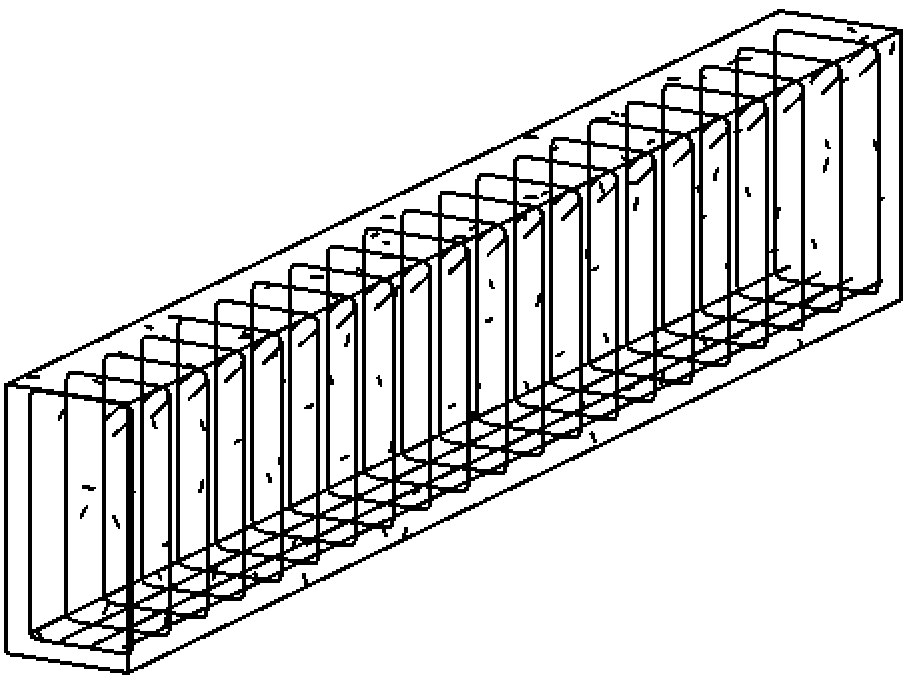
$$c=\frac{a}{\beta1}=\frac{102}{0.836}=122\ mm$$

$$\epsilon t=\frac{dt-c}{c}\epsilon u=\frac{368-122}{122}*0.003=6.05*10^{-3}>0.005$$

$\Phi=0.9$ O.K

9. Draw final detailed section


300 mm
430 mm
Φ10 mm
Stirrups
40 mm
40 mm
4φ25



Impact of Computers on Reinforced Concrete Design

The availability of personal computers has drastically changed the way in which reinforced concrete structures are analyzed and designed. In nearly every engineering school and office, computers are routinely used to handle structural design problems.

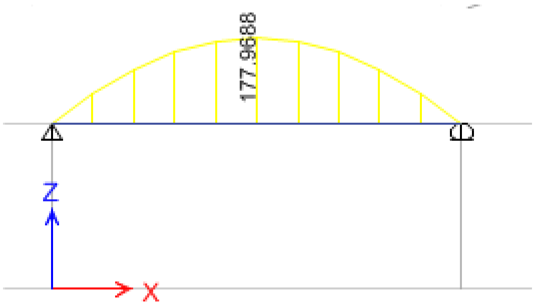
Many calculations are involved in reinforced concrete design, and many of these calculations are quite time consuming. With a computer, the designer can reduce the time required for these calculations tremendously.

Although computers do increase design productivity, they do undoubtedly tend at the same time to reduce the designer's "feel" for structures. This can be special problem for young engineers with little previous design experience.

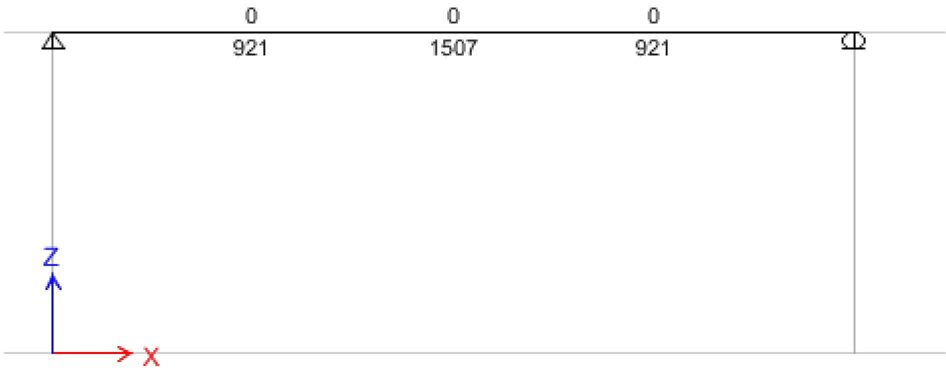
Resolve the previous example by using any concrete design software

- ETABS 2015 has been adopted to model the previous example.
- ETABS is a structural engineering software , ETABS was used to create the mathematical model of the **Burj Khalifa**, currently the world's tallest building
- Good agreements have been found when compare the results with hand calculations.

Moment diagram



Area of steel (As) mm^2



Material Properties

E_c (MPa)	f'_c (MPa)	Lt.Wt Factor (Unitless)	f_y (MPa)	f_{ys} (MPa)
24855.58	30	1	400	400

Design Code Parameters

Φ_T	Φ_{CTied}	$\Phi_{CSpiral}$	Φ_{Vns}	Φ_{Vs}	Φ_{Vjoint}
0.9	0.65	0.75	0.75	0.6	0.85

Flexural Reinforcement for Moment, M_{u3}

	Required Rebar mm²	+Moment Rebar mm²	-Moment Rebar mm²	Minimum Rebar mm²
Top (+2 Axis)	0	0	0	0
Bottom (-2 Axis)	1507	1507	0	380

Design Moments, M_{u3}

Design +Moment kN-m	Design -Moment kN-m
177.9688	0

Method for analysis	Max Moment	Area of steel (As)	Difference %
Ultimate strength method	177.97 kN.m	1501.44 mm^2	0
ETABS Solutions	177.97 kN.m	1507 mm^2	0.37 %

Example2: Design a rectangular beam to support a dead load of 35 kN/m and a live load of 25 kN/m acting on a simple span 6m. Assume that the designer intends to use:

- A width of 300mm and a depth of 700 mm.
- *fc*`=21 Mpa and *fy*=420 Mpa.
- Bar diameter of 25mm for longitudinal reinforcement.
- One layer of reinforcement.
- Bar diameter of 10mm for stirrups.

Solution:

1. Compute required factored applied moment **Mu**

$W_{selfweight}$=0.3*0.7*24=5.04 kN/m

W_{dead}=35+5.04=40.04 kN/m

Wu=$1.2W_D+1.6W_L$=1.2*40.04+1.6*25=88.048 kN/m

$M_u = \frac{Wu\ell^2}{8} = \frac{88.048*6^2}{8}$ =396 kN.m

2. Compute the effective depth (d)

d $_{\text{for one layer}}$**=h-cover-stirrups-**$\frac{\boldsymbol{bar\ diameter}}{\mathbf{2}}$

d=700-40-10- $\frac{\mathbf{25}}{\mathbf{2}}$ =637.5 mm

3. Compute ρ_{max}, ρ_{min}

$\rho_{max} = 0.85\beta_1 \frac{fc`}{fy} \frac{\epsilon u}{\epsilon u + 0.004}$ and $\epsilon\ u = 0.003$

$\rho_{max} = 0.85*0.85*\frac{21}{420}\frac{0.003}{0.003+0.004} = 15.5*10^{-3}$

$\rho_{min} \frac{1.4}{fy} = \frac{1.4}{420} = 3.33*10^{-3}$

4. Compute ρ

$\rho = \frac{1}{\mathrm{m}}(1-\sqrt{1-\frac{2\mathrm{Rm}}{\mathrm{fy}}})$

$\mathrm{m} = \frac{fy}{0.85fc`} = \frac{420}{0.85*21} = 23.5$

$\mathrm{R} = \frac{M_u*10^6}{\varphi b d^2} = \frac{396*10^6}{0.9*300*637.5^{\ 2}} = 3.61$

$\rho = \frac{1}{23.5}(1-\sqrt{1-\frac{2*3.61*23.5}{420}}) = 9.7*10^{-3}$

$\rho_{min} = 3.33*10^{-3} < \rho = 9.7*10^{-3} < \rho_{max} = 15.5*10^{-3}$ O.K

5. Compute the required steel area (As):

   $As = \rho * b * d = 9.7*10^{-3}*300*637.5 = 1855.13\ mm^2$

6. Compute required Number of rebars:

   $$\text{No. of rebars} = \frac{As}{Abar} = \frac{1855.13}{\frac{\pi}{4}*25^2} = \frac{1855.13}{491} = 3.7 \approx 4$$

   Try 4φ25mm.

   $As_{provided} = 4*491 = 1964\ mm^2$

7. Check if the available width "b" is adequate to put the rebars in a single layer:

   $b_{required}$ = 2 x cover+2 x stirrups diameter + No. of rebars x bar diameter + (No. of rebars-1) x spacing between rebars

   $b_{required} = 2*40+2*10+4*25+3*25 = 275\ mm < 300\ mm$ O.K

8. Check the assumption of φ=0.9

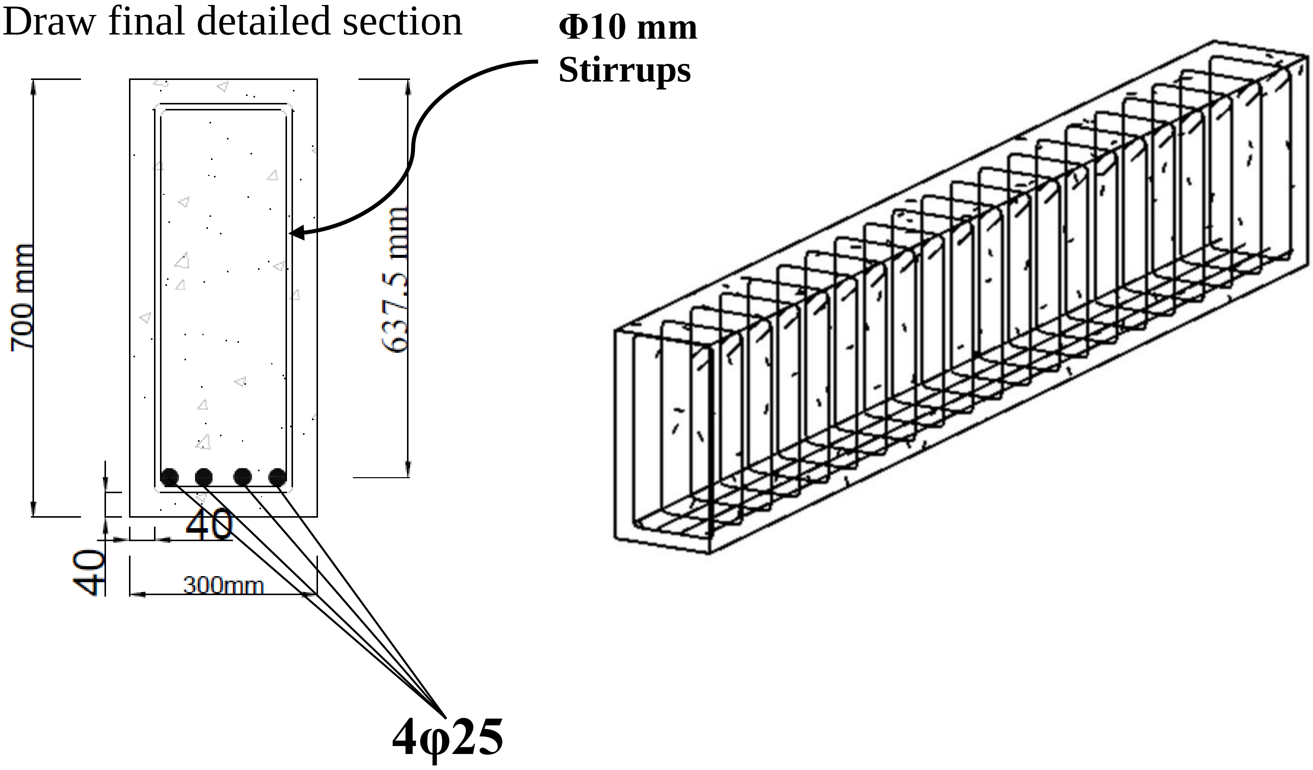
   $$a = \frac{As*fy}{0.85fc`*b} = \frac{1964*420}{0.85*21*300} = 154\ mm$$

   $$c = \frac{a}{\beta1} = \frac{154}{0.85} = 181\ mm$$

   $$\epsilon t = \frac{dt-c}{c}\epsilon u = \frac{637-181}{181}*0.003 = 7.56*10^{-3} > 0.005$$

   Φ=0.9 O.K

9. Draw final detailed section



Example3: Design a simply supported beam with a span of 6.1 m to support following loads:

- A dead load of 8.22 kN/m
- A live load of 24.1 kN/m
- Concrete of fc\`=35 Mpa
- Steel of fy=420 Mpa
- A width of 325mm and a height of 420mm.
- Bar diameter of 25mm for longitudinal reinforcement.
- Bar diameter of 12mm for stirrups.
- Two layers of reinforcements.

Solution:

1. Compute required factored applied moment **Mu**

$W_{selfweight}$=0.325*0.42*24=3.28 kN/m

W_{dead}=8.22 +3.28 =11.5 kN/m

Wu=1.2W_D+1.6W_L=1.2*11.5 +1.6*24.1 =52.36 kN/m

$M_u = \frac{Wu\ell^2}{8} = \frac{52.36*6.1^2}{8}$ =243 kN.m

2. Compute the effective depth (d)

$\mathbf{d}_{\textbf{for two layer}}$**=h-cover-stirrups-bar diameter-**$\frac{\boldsymbol{spacing\ between\ layers}}{\mathbf{2}}$

d=420-40-12-20- $\frac{\mathbf{25}}{\mathbf{2}}$ **=**335.5 mm

3. Compute ρ_{max}, ρ_{min}

$\rho_{max} = 0.85\beta_1 \frac{fc`}{fy} \frac{\epsilon u}{\epsilon u + 0.004}$ and $\epsilon\ u$ =0.003

$\beta_1 = 0.85 - \frac{0.05(35-28)}{7} = 0.8$

$\rho_{max} = 0.85*0.8*\frac{35}{420}\frac{0.003}{0.003+0.004} = 24*10^{-3}$

$\rho_{min} = \frac{1.4}{fy}\ or\ \frac{\sqrt{fc`}}{4fy} = \frac{1.4}{420}\ or\ \frac{\sqrt{35}}{4*420} = 3.33*10^{-3}$ or **$\underline{3.52*10^{-3}}$** (use larger)

4. Compute ρ

$\rho = \frac{1}{\mathrm{m}}(1 - \sqrt{1 - \frac{2\mathrm{Rm}}{\mathrm{fy}}})$

$\mathrm{m} = \frac{fy}{0.85fc`} = \frac{420}{0.85*35} = 14.12$

$\mathrm{R} = \frac{M_u*10^6}{\varphi bd^2} = \frac{243*10^6}{0.9*325*335.5^2} = 7.38$

$\rho = \frac{1}{14.12}(1-\sqrt{1-\frac{2*7.38*14.12}{420}}) = 20.55*10^{-3}$

$\rho_{min}=3.52*10^{-3} < \rho=20.55*10^{-3} < \rho_{max}=24*10^{-3}$ O.K

5. Compute the required steel area (As):

   $As = \rho*b*d = 20.55*10^{-3}*325*334 = 2231\ mm^2$

6. Compute required Number of rebars:

   No. of rebars $=\frac{As}{Abar} = \frac{2231}{\frac{\pi}{4}*20^2} = \frac{2231}{314} = 7.1 \approx 8$

   Use 8φ20

   $As_{provided} = 8*314 = 2512\ mm^2$

7. Check if the available width "b" is adequate to put the rebars in a single layer:

   $b_{required}$ = 2 x cover+2 x stirrups diameter + No. of rebars x bar diameter + (No. of rebars-1) x spacing between rebars

   $b_{required}=2*40+2*12+4*20+3*25=259\ mm < 325\ mm$ O.K

8. Check the assumption of φ=0.9

   $a = \frac{As*fy}{0.85fc`*b} = \frac{2512*420}{0.85*35*325} = 109\ mm$

   $c = \frac{a}{\beta1} = \frac{109}{0.8} = 136\ mm$

   $\epsilon t = \frac{dt-c}{c}\epsilon u = \frac{358-136}{136}*0.003 = 4.89*10^{-3} < 0.005$

   as $\epsilon t < 0.005$

   $\varphi = 0.483+83.3*\epsilon t = 0.483+83.3*4.89*10^{-3} = 0.89$

   - Reinforcement Re-design based on new φ

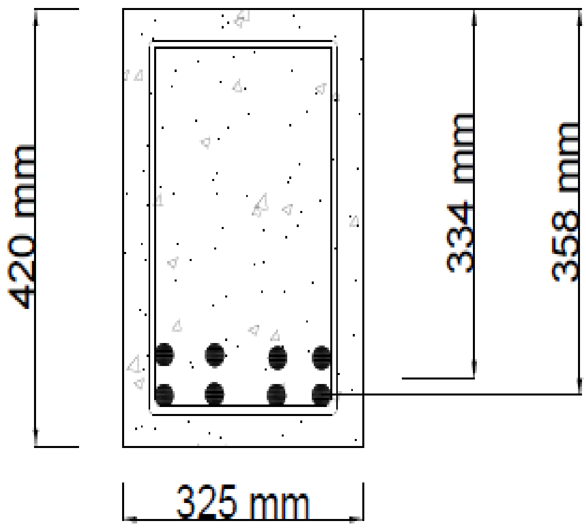
   Retain to step 4 and calculate new steel ratio ρ

   $\rho=20.83*10^{-3}$

   $As = \rho*b*d = 20.83*10^{-3}*325*334 = 2261.1\ mm^2$

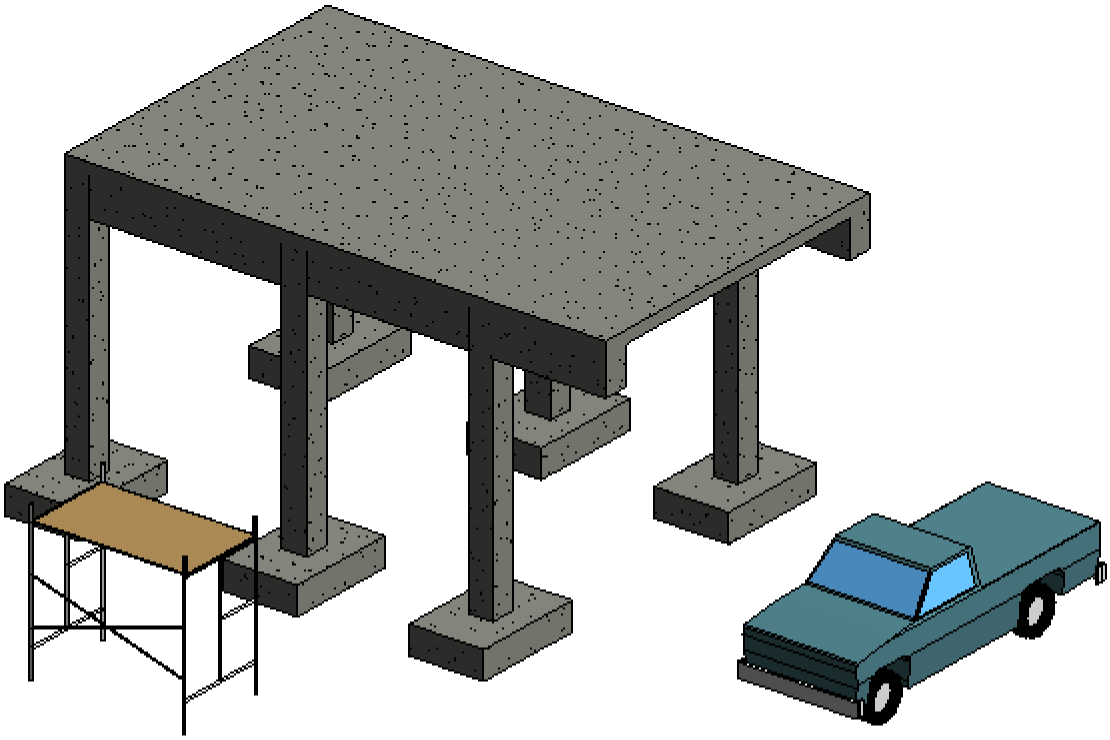
   No. of rebars $=\frac{As}{Abar} = \frac{2261.1}{\frac{\pi}{4}*20^2} = \frac{2261.1}{314} = 7.2 \approx 8$

9. Draw final detailed section



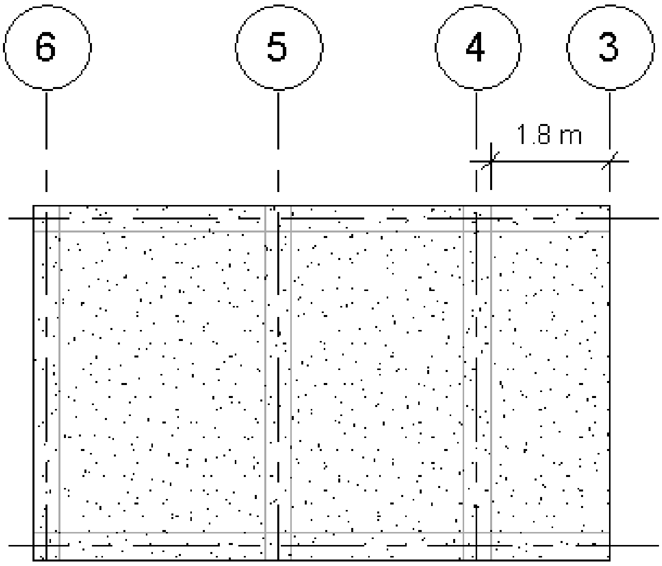
Example 4: Design a cantilever beam shown in figure below to carry the following loads:

- A dead load=50 kN/m (include self-weight)
- A live load=71.25 kN/m



Assume that the designer intends to use:

- Concrete fc`=28 Mpa
- Steel fy=420 Mpa
- A width of 400 mm and a height of 800mm for cantilever.
- Bar diameter 25mm for longitudinal reinforcement.
- Bar diameter 12mm for stirrups.
- Single layer of reinforcement.



Solution:

1. Compute required factored applied moment **Mu**

   W_{dead}=50 kN/m

   Wu=1.2W_D+1.6W_L=1.2*50+1.6*71.25 =174 kN/m

   $M_u= \frac{Wu\ell^2}{2} = \frac{174*1.8^2}{2}$ =282 kN.m

2. Compute the effective depth (d)

   d $_{\textbf{for one layer}}$**=h-cover-stirrups-**$\frac{\boldsymbol{bar\ diameter}}{\mathbf{2}}$

   d=800-40-12- $\frac{\mathbf{25}}{\mathbf{2}}$ =735.5 mm

3. Compute ρ_{max}, ρ_{min}

   ρ_{max} =0.85$\beta_1 \frac{fc`}{fy} \frac{\epsilon u}{\epsilon u+0.004}$ and $\epsilon\ u$ =0.003

   ρ_{max} =0.85*0.85$\frac{28}{420} \frac{0.003}{0.003+0.004}$=20.6*$10^{-3}$

   $\rho_{min}=\frac{1.4}{fy}=\frac{1.4}{420}$ =3.33*10^{-3}

4. Compute ρ

   $\rho= \frac{1}{m}(1- \sqrt{1-\frac{2Rm}{fy}})$

   $m=\frac{fy}{0.85fc`} =\frac{420}{0.85*28}$ =17.6

   $R= \frac{M_u*10^6}{\varphi bd^2} =\frac{282*10^6}{0.9*400*735.5^2}$ =1.448

   $\rho= \frac{1}{17.6}(1- \sqrt{1-\frac{2*1.448*17.6}{420}})$ =3.56*10^{-3}

   ρ_{min}=3.33*10^{-3} < ρ=3.56*10^{-3}< ρ_{max} =20.6*10^{-3} O.K

5. Compute the required steel area (As):

   As= $\rho*b*d$=3.56*10^{-3}*400*735.5 =1047 mm^2

6. Compute required Number of rebars:

   No. of rebars=$\frac{As}{Abar} =\frac{1047}{\frac{\pi}{4}*25^2}=\frac{1047}{490}$ =2.13≈ 3

   Try 3φ20 mm

   $As_{provided}$ =3*490=1470 mm^2

7. Check if the available width "b" is adequate to put the rebars in a single layer:

   $b_{required}$ = 2 x cover+2 x stirrups diameter + No. of rebars x bar diameter + (No. of rebars-1) x spacing between rebars

   $b_{required}$=2*40+2*12+3*25+2*25=229 mm < 400 mm O.K

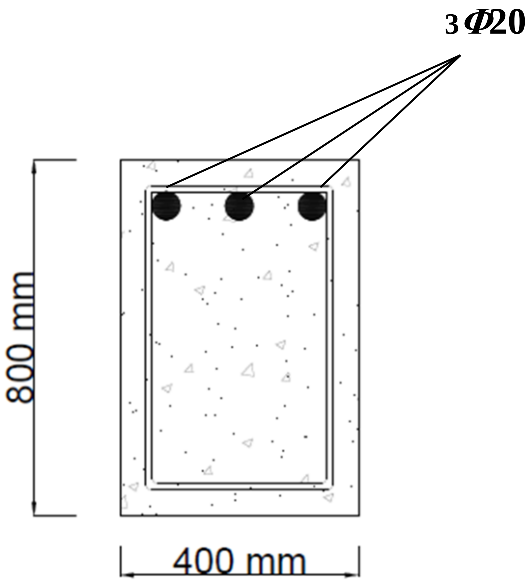
8. Check the assumption of φ=0.9

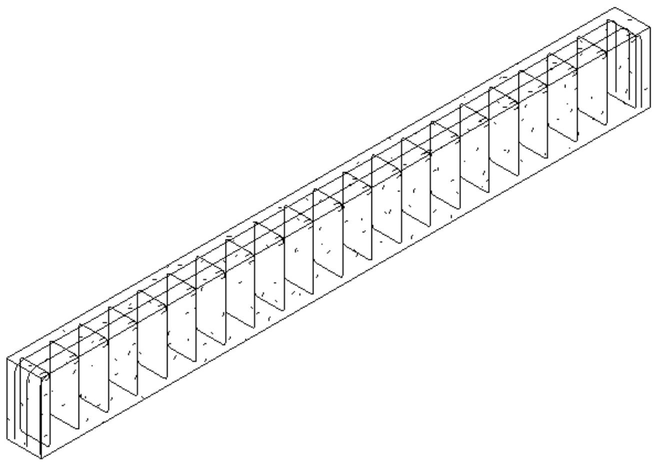
   $a = \frac{As*fy}{0.85fc`*b} = \frac{1470*420}{0.85*28*400} = 64.9$ mm

   $c = \frac{a}{\beta 1} = \frac{64.9}{0.85} = 76.3$ mm

   $\epsilon t = \frac{dt-c}{c}\epsilon u = \frac{735.5-76.3}{76.3}*0.003 = 0.026 > 0.005$

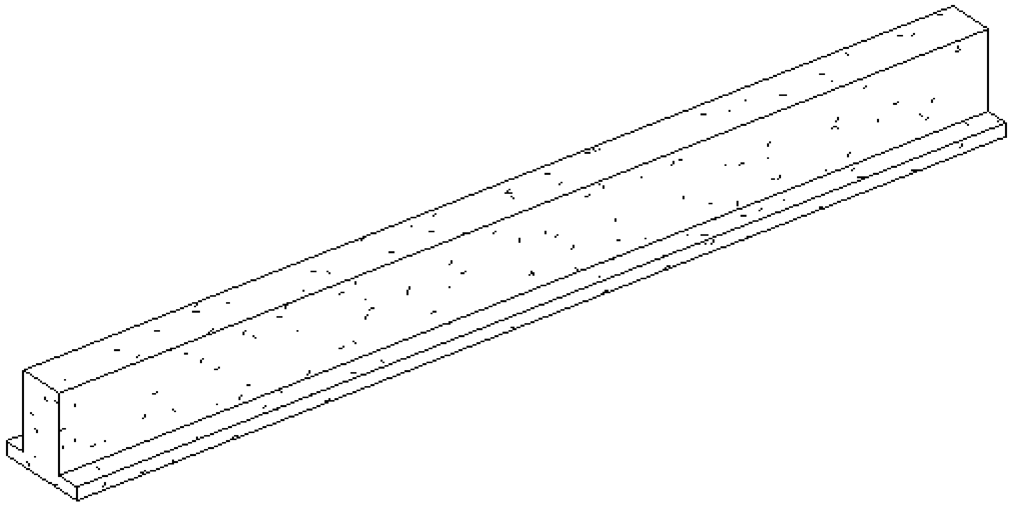
   Φ=0.9 O.K

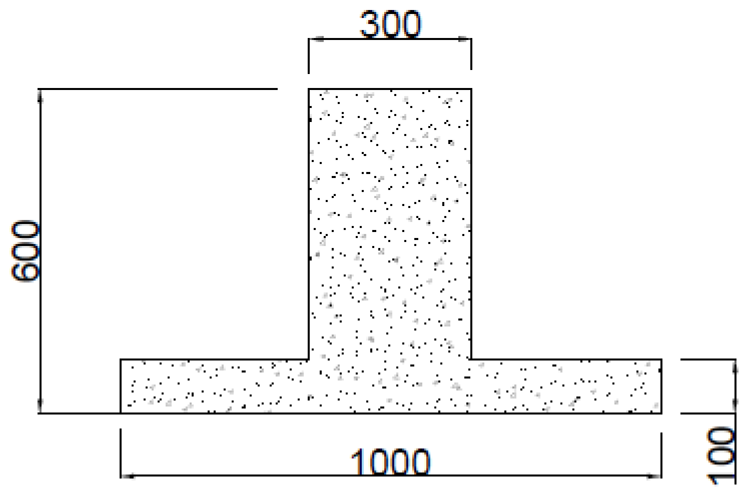




Example 5: Design the beam shown in figure below assume:

- The beam in simply supported and the length of beam=6m.
- W_{dead}=15 kN/m W_{live}=12 kN/m
- fc`=28 Mpa , fy=420 Mpa
- Bar diameter of 25mm longitudinal reinforcement.
- Bar diameter of 10mm for stirrups.
- Single layer of reinforcement.





Solution:

$W_{selfweight}$=0.1*1*24+0.5*0.3*24=6 kN/m

W_{dead}=6+15=21 kN/m

Wu=$1.2W_D+1.6W_L$=1.2*21+1.6*12=44.4 kN/m

Mu=$\frac{Wu\ell^2}{8} = \frac{44.4*6^2}{8}$ =200 kN.m

$d_{for\ one\ layer}$=600-40-10-12.5=538 mm

$\rho_{max} = 0.85\beta_1 \frac{fc`}{fy} \frac{\in u}{\in u + 0.004}$ and $\in u$ =0.003

ρ_{max} =20.6*10^{-3}

Compute ρ

$\rho = \frac{1}{m}(1- \sqrt{1 - \frac{2Rm}{fy}})$

ρ=6.46*10^{-3}

As= $\rho*b*d$=6.46*10^{-3}*300*538=1042 mm^2

No. of bars=2.12≈3

Try 3φ25 mm

As provided=3*491=1470 mm^2

$b_{required}$=2*40+2*10+3*25+2*25=225 mm < 300mm O.K